[10-09-15-T12]

CC: [3.10] #1 is a loser

■ Note:

$P(A) = 0.a = \frac{a}{100} = \frac{n(A)}{n(S)} \Longrightarrow a = n(A),\ n(S) = 100.$

So, for example, $P(A) = 0.59 = \frac{59}{100} = \frac{n(A)}{n(S)} \Longrightarrow n(A) = 59,\ n(S) = 100.$

In the following, C names the set of people in the sample who drink coffee, T the set of people in the sample who eat toast. $\overline{C}$ and $\overline{T}$ are the complements of C and T respectively. S is the sample space.

■ Given in statement of problem.

$P(C) = \frac{89}{100}$ means 89 elements in C, sample space S size 100.

$P(T) = \frac{69}{100}$ means 69 elements in T, sample space S size 100.

$P\left(C \text{ and } T\right) = \frac{73}{100}$ means 73 elements in (C and T), sample space S size 100.

	T	$\overline{T}$	
C	73		89
$\overline{C}$			
	69		100

■ Derived, using the Law of Non-contradiction: $\neg (p \wedge \neg p)$ and the axioms of probability theory.

$P(\overline{C}) = 1 - \frac{89}{100} = \frac{11}{100}$, because C and $\overline{C}$ are mutually exclusive.

$P(\overline{T}) = 1 - \frac{69}{100} = \frac{31}{100}$, because T and $\overline{T}$ are mutually exclusive.

$P\left(C \text{ and } T\right) = \frac{73}{100}$

Thus far,

	T	$\overline{T}$	
C	73		89
$\overline{C}$			**11**
	69	**31**	100

Since T and $\overline{T}$ are mutually exclusive even for non coffee-drinkers, 73 + **16** = 89.

	T	$\overline{T}$	
C	73	**16**	89
$\overline{C}$			**11**
	69	**31**	100

Similarly, C and $\overline{C}$ are mutually exclusive even for non toast-eaters, $16 + \mathbf{15} = 31$.

	T	$\overline{T}$	
C	73	**16**	89
$\overline{C}$		**15**	**11**
	69	**31**	100

Well, C and $\overline{C}$ are mutually exclusive even toast-eaters, $16 + \mathbf{(-4)} = 69$.

	T	$\overline{T}$	
C	73	**16**	89
$\overline{C}$	**-4**	**15**	**11**
	69	**31**	100

You mean there are negative 4 people who do not drink coffee and do eat toast?
— Well, of course I do not mean it. I can't. Because it is meaningless!

But note that the conclusion that there are negative 4 people who do not drink coffee but do eat toast is *forced* on us by the numbers given in the statement of the problem together with the Law of Non-contradiction and the axioms of probability theory. So, either we must allow that negative 4 people don't drink coffee and do eat toast *or* toss out the Law of Non-contradiction *or* denie at least some of the axioms of probability *or* reject the numbers given in the statement of the problem.

■ Conclusion

Of course, the Law of Non-contradiction is unassailable. So, one who wishes to maintain that the numbers given in the problem ought to be kept owes us an explanation of what meaning the utterance "Negative 4 people do not drink coffee but do eat toast" might possibly have *or* must reject at least an axiom of probability theory.

■ Question

Is S a probability space?